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where $\alpha, \beta, \gamma, \dots$ are the six different prime factors 2, 3, 5, 7, 31, 61 (with repetition 2 and 5) combined in products of 1, 2, 3, ..., 10 letters at a time.

By actual calculation and checking we have found the following ten solutions:

$$x = -3343, 12721, 320, 305, 7547, -2225, -710, -3215, -8495, -56950;$$

$$y = 2, 8, 29, 374, 4187, -1, -91, -1486, -4276, -29851;$$

$$u = 568, -2634, -116, 454, 5923, +337, -126, -2136, -6102, -42446;$$

$$v = -137, -89, -101, 319, 5473, -141, -261, -2121, -5841, -39941.$$

Also solved by G. B. M. Zerr.

169. Proposed by R. D. CARMICHAEL, Princeton University.

Let $Q_n(x)=0$ be the equation whose roots are all the primitive n th roots of unity without repetition. In $Q_n(x)=0$ replace x by α/β , a fraction in its lowest terms, and clear of fractions. Let $Q_n(\alpha, \beta)$ represent the resulting first member. Set $n=mp$ where p is the largest prime factor of n . It is required to find all the integral values of α, β, m, p satisfying the following relations:

$$\begin{aligned} (1) \quad & Q_{mp}(\alpha, \beta) = p, \\ (2) \quad & \alpha^m - \beta^m \equiv 0 \pmod{p}. \end{aligned}$$

One such solution is: $\alpha=2, \beta=1, m=2, p=3$. (See MONTHLY, Vol. XII, p. 89.)

[No solution of this problem has been received.]

170. Proposed by PATRICK WALSH, 1451 Annunciation Street, New Orleans, La.

The areas of rectangles A and B are respectively $15170 \frac{10}{27}$ and 31230.3627 . Find the sides and diagonal of each rectangle in exact or rational numbers.

Solution by B. F. FINKEL, Ph. D.

For A , let x and y be the dimensions of the field. Then

$$xy = 15170 \frac{10}{27} = \frac{2^{14} \cdot 5^3}{3^3} \dots (1), \text{ and } \sqrt{x^2 + y^2} = d \dots (2),$$

where d is the diagonal, which is to be rational. Solving (1) for y and substituting the value thus found in (2) and reducing, we have

$$\sqrt{\frac{3^6 x^4 + 2^{28} \cdot 5^4}{3^3 y}} = d.$$

Let $x = \frac{2^7 \cdot 5}{3^3} z$. Then $d = \frac{2^7 \cdot 5}{3^2 z} \sqrt{z^4 + 9}$. Let $\sqrt{z^4 + 9} = z^2 t - 3$. Then

$$z^2 = \frac{6t}{t^2 - 1}.$$

We must now find such values for t as will make $\frac{6t}{t^2-1}$ a perfect square. These values are 0, 1, 2, 3, $\frac{1}{2}$, $\frac{1}{3}$. For $t=2$, $z=2$,

$$d=\frac{2^6 \cdot 5^2}{3^2}, x=\frac{2^8 \cdot 5}{3^2}, \text{ and } y=\frac{2^6 \cdot 5}{3}.$$

For $t=3$, $z=\frac{3}{2}$, $d=\frac{2^6 \cdot 5^3}{3^2}$, $x=\frac{2^6 \cdot 5}{3}$, and $y=\frac{2^8 \cdot 5^2}{3^2}$.

The values of 0 and 1 for t , give reciprocal limiting values for d , x , and y .

A similar treatment for B leads to the value, $z=2\sqrt{\frac{15t}{t^2-1}}$.

z is rational for $t=0, 1, 4, -\frac{1}{4}$, giving the values $z=0, \infty, 4$.

The values of x and y corresponding to $z=4$, are

$$x=\frac{3^2 \cdot 19 \cdot 179}{2^3 \times 5}, y=\frac{3 \times 19 \cdot 179}{5^2}, \text{ diagonal}=\frac{3^2 \cdot 17 \cdot 19 \cdot 179}{2^3 \times 5}.$$

171. Proposed by PROFESSOR E. B. ESCOTT, Ann Arbor, Mich.

Solve completely:

$$\begin{aligned} 2x^2-1 &= y, \\ 2y^2-1 &= z, \\ 2z^2-1 &= w, \\ 2w^2-1 &= x. \end{aligned}$$

I. Solution by PROFESSOR L. E. DICKSON, Ph. D., The University of Chicago.

It will be shown that the only integral solution is $x=y=z=w=1$.

If $x=0$ or ± 1 , then $y^2=1$, $z=1$, $w=1$, so that the fourth equation gives $x=1$.

Next, let $x^2 > 1$. Then $y > x^2$, $z > y^2 > 1$, $w > z^2 > 1$, $x > w^2$.

Hence, $x > x^{16}$, which contradicts $x^2 > 1$.

Also similarly solved by G. B. M. Zerr, and S. G. Barton.

II. Solution by the PROPOSER.

Let $x=\cos\phi$; then $y=\cos 2\phi$, $z=\cos 4\phi$, $w=\cos 8\phi$, $x=\cos 16\phi$.

Since $\cos 16\phi = \cos\phi$, we have, $\cos 16\phi - \cos\phi = 0$, which may be written

$$-2\sin^{\frac{1}{2}7}\phi \cdot \sin^{\frac{1}{2}5}\phi = 0.$$

If $\sin^{\frac{1}{2}7}\phi = 0$, $\phi = \frac{2n\pi}{17}$. If $\sin^{\frac{1}{2}5}\phi = 0$, $\phi = \frac{2n\pi}{15}$.